RUL prediction for two-phase degradation model based on reparameterized inverse Gaussian process

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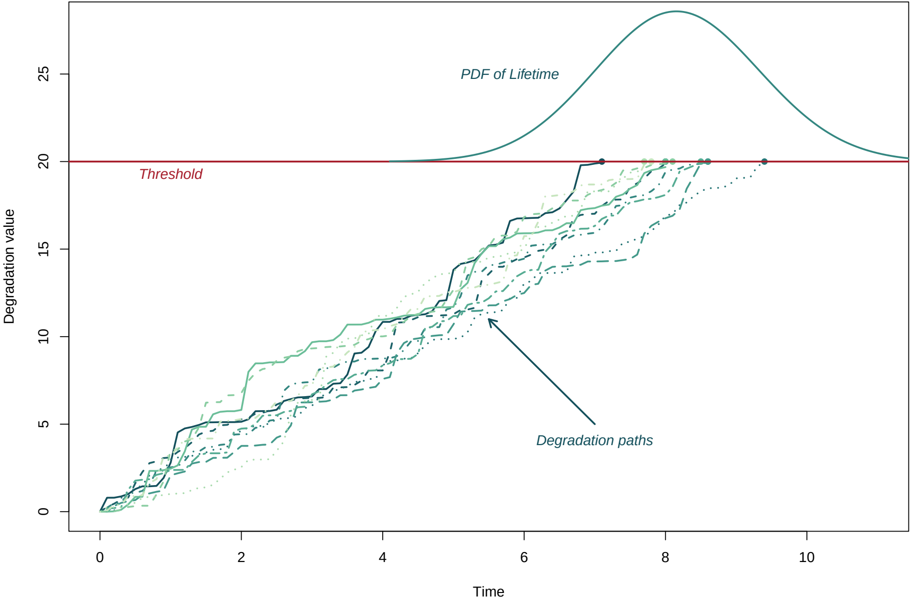
Outline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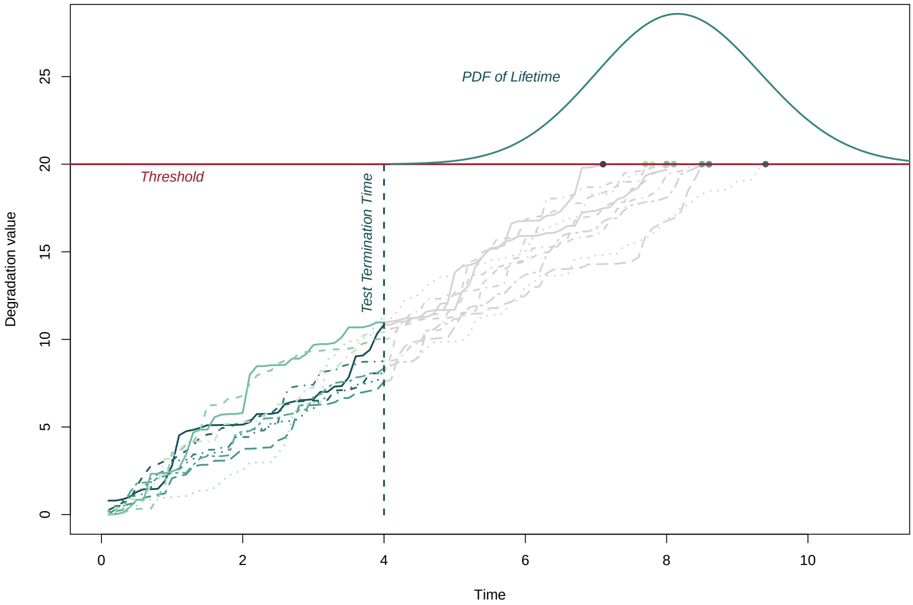
PDF of Lifetime
Threshold
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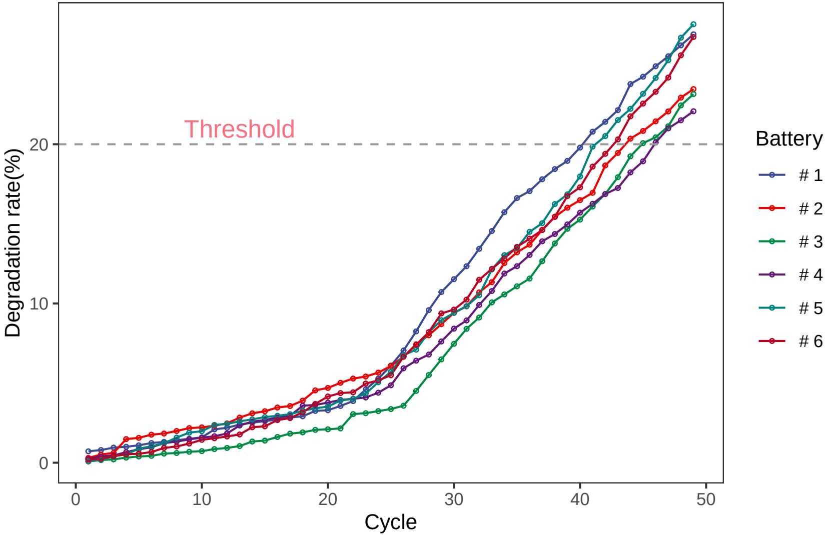
Degradation Models

- General path model.
- Stochastic process: Wiener, gamma, inverse Gaussian (IG), variance gamma, Ornstein–Uhlenbeck, etc.
- Review papers: Si et al. (2011), Ye and Xie (2015), Zhang et al. (2018).

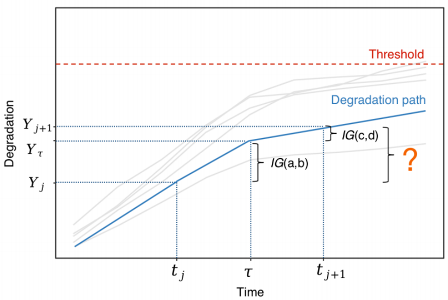
Two-stage degradation



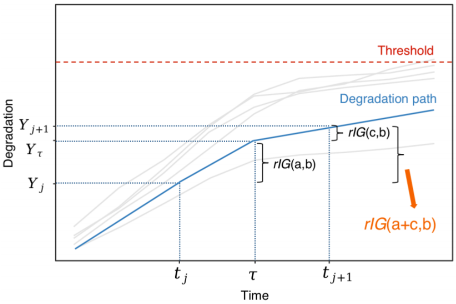
Related Literature

Two-phase degradation modeling

1. Wiener process: Wang et al. (2018a, 2018b), Zhang et al. (2019), Lin et al. (2021), Ma et al. (2023), etc.
2. Gamma process: Ling et al. (2019), Lin et al. (2021).
3. Inverse Gaussian (IG) process: Duan and Wang (2017).
   - Limitations of Duan and Wang (2017):
     - (i) Constraints on locations of change points;
     - (ii) Insufficient considerations for deriving the lifetime distribution;
     - (iii) Neglecting the uncertainty in estimation.



(a) IG process



(b) Re-parameterized IG process

Contributions

(i) A novel two-phase reparameterized IG (rIG) degradation model with distinct change points and model parameters for each individual system;

(ii) Derive the distribution of failure time and RUL, and propose an adaptive replacement policy;

(iii) Employ bootstrap and Bayesian approach to generate interval estimates for the parameters.

Outline

Reparameterized IG (rIG) distribution

Probability density function (PDF)

If a random variable Y follows RIG distribution, then its PDF is

$$f_{rIG}(y|\delta,\gamma) = \frac{\delta}{\sqrt{2\pi}} e^{\delta\gamma} y^{-3/2} e^{-\left(\delta^2 y^{-1} + \gamma^2 y\right)/2}, \; y > 0, \; \delta > 0, \; \gamma > 0. \qquad (1)$$

Denoted as $Y \sim rIG(\delta,\gamma)$.

Cumulative distribution function (CDF)

$$F_{rIG}(y|\delta,\gamma) = \Phi\left[\sqrt{y}\gamma - \frac{\delta}{\sqrt{y}}\right] + e^{2\delta\gamma}\Phi\left[-\sqrt{y}\gamma - \frac{\delta}{\sqrt{y}}\right], \qquad (2)$$

where $\Phi(\cdot)$ is the CDF of the standard normal distribution.

Moment generating function (MGF)

$$M_Y(t) = E(e^{ty}) = e^{\delta\gamma\left(1-\sqrt{1-\frac{2t}{\gamma^2}}\right)}. \tag{3}$$

Additive property

If $Y_1 \sim rIG\left(\delta_1, \gamma\right), Y_2 \sim rIG\left(\delta_2, \gamma\right)$, then $Y_1 + Y_2 \sim rIG\left(\delta_1 + \delta_2, \gamma\right).$

rIG process

Definition of rIG process

rIG process $\{Z(t), t \geq 0\}$ satisfies the following properties:

(i) $Z(0) = 0$ with probability one;

(ii) $Z(t)$ has independent increments. Specifically, $Z(t_2) - Z(t_1)$ and $Z(s_2) - Z(s_1)$ are independent for all $t_2 > t_1 \geq s_2 > s_1 \geq 0$;

(iii) For all $t > s \geq 0$, $Z(t) - Z(s)$ follows the rIG distribution $rIG(\delta(\Lambda(t) - \Lambda(s)), \gamma)$, where $\Lambda(t)$ is a monotone increasing function with $\Lambda(0) = 0$, δ and γ are unknown parameters.

- Denoted as $r\mathcal{IG}(\delta\Lambda(t), \gamma)$.
- The mean and variance of $\{Z(t), t \geq 0\}$, which are $\delta\Lambda(t)/\gamma$ and $\delta\Lambda(t)/\gamma^3$, respectively.

Two-phase rIG degradation model

Two-phase rIG degradation model

Suppose a system's performance characteristic degrades in two distinct phases, separated by a single change point.

$$\begin{aligned} & Y(t)|\tau \sim r\mathcal{IG}\left(m(t;\delta_1,\delta_2,\tau),\gamma\right),\ \tau \sim N\left(\mu_\tau,\sigma_\tau^2\right), \\ & m(t;\delta_1,\delta_2,\tau) = \begin{cases} \delta_1 t, & t \le \tau, \\ \delta_2\left(t-\tau\right)+\delta_1\tau, & t > \tau, \end{cases} \end{aligned} \tag{4}$$

where δ_1 and δ_2 are the drift parameters for $t \le \tau$ and $t > \tau$, respectively.

Failure-time

Let $T = \inf\{t \mid Y(t) \geq \mathcal{D}\}$, and $Y(t) = \begin{cases} Y_1(t), & t \leq \tau, \\ Y_1(\tau) + Y_2(t-\tau), & t > \tau. \end{cases}$

Conditional reliability function of T

- $0 \leq t \leq \tau$

$$\bar{F}_1(t \mid \tau) = P(T > t \mid \tau \geq t) = P(Y_1(t) < \mathcal{D} \mid \tau \geq t) = F_{r\mathcal{IG}}(\mathcal{D}|\delta_1 t, \gamma). \tag{5}$$

- $t > \tau$

$$\begin{aligned} \bar{F}_2(t \mid \tau) =& P\left(Y(t) < \mathcal{D} \mid \tau < t\right) = P\left(Y_1(\tau) + Y_2(t-\tau) < \mathcal{D} \mid \tau < t\right) \\ =& \int_0^{\mathcal{D}} F_{r\mathcal{IG}}(\mathcal{D} - y_\tau|\delta_2(t-\tau), \gamma) f_1(y_\tau \mid \tau) \mathrm{d}y_\tau, \end{aligned} \tag{6}$$

where y_τ represents the degradation value at τ, and $f_1(y_\tau \mid \tau)$ is the PDF of y_τ.

Failure-time

Unconditional reliability function of T

$$\begin{aligned} R(t) &= P\left(Y(t) < \mathcal{D}, \tau \geq t\right) + P\left(Y(t) < \mathcal{D}, 0 < \tau < t\right) \\ &= \bar{F}_1\left(t \mid \tau\right) \bar{G}_\tau(t) + \int_0^t g_\tau(\tau|\mu_\tau, \sigma_\tau^2) \bar{F}_2\left(t \mid \tau\right) d\tau, \end{aligned} \tag{7}$$

where $\bar{G}_\tau(t)$ is the survival function of random variable τ.

Mean time to failure (MTTF)

$$\text{MTTF} = E(T) = \int_0^\infty R(t)dt. \tag{8}$$

RUL

Let $S_t = \inf\{x; Y(t+x) \geq \mathcal{D} \mid Y(t) < \mathcal{D}\}$.

Conditional reliability function of S_t

(i) When $x + t \leq \tau$:

$$\bar{F}_{S_t,1}(x \mid \tau) = F_{r\mathcal{IG}}(\mathcal{D} - Y(t) | \delta_1 x, \gamma). \tag{9}$$

(ii) When $t < \tau < x + t$:

$$\begin{aligned}\bar{F}_{S_t,2}(x \mid \tau) &= P(Y(t+x) < \mathcal{D} \mid Y(t) \leq \mathcal{D}) \\ &= \int_0^{\mathcal{D}} F_{r\mathcal{IG}}(\mathcal{D} - y_\tau | \delta_2(t + x - \tau), \gamma) f_1(y_\tau \mid \tau) \mathrm{d}y_\tau.\end{aligned} \tag{10}$$

(iii) When $\tau \leq t$:

$$\bar{F}_{S_t,3}(x \mid \tau) = F_{r\mathcal{IG}}(\mathcal{D} - Y(t) | \delta_2 x, \gamma). \tag{11}$$

Remaining useful life (RUL)

Unconditional reliability function of S_t

$$\begin{aligned} R_{S_t}(x) =& P(Y(t+x) < \mathcal{D}, t < x+t \leq \tau) \\ & + P(Y(t+x) < \mathcal{D}, t \leq \tau < x+t) + P(Y(t+x) < \mathcal{D}, t > \tau) \\ =& \bar{F}_{S_t,1}(x \mid \tau)\, \bar{G}_\tau(x+t) + \int_t^{x+t} g_\tau(\tau|\mu_\tau, \sigma_\tau^2) \bar{F}_{S_t,2}(x \mid \tau)\, d\tau \\ & + \int_0^t g_\tau(\tau) \bar{F}_{S_t,3}(x \mid \tau)\, d\tau. \end{aligned} \tag{12}$$

Mean of RUL at time t

$$\text{MRL} = E(S_t) = \int_0^\infty R_{S_t}(x) dx. \tag{13}$$

Outline

Data

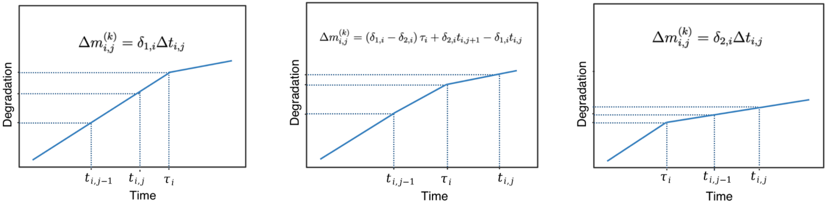
- I systems under inspection in a degradation test.
- Deterioration pattern follows the two-phase rIG degradation model.
- $Y_{i,j}$ is the observed degradation value at the measurement time $t_{i,j}$, $i = 1 \ldots, I,\ j = 1, \ldots, n_i$, and $0 < t_{i,1} < \ldots < t_{i,n_i}$.
- Let $\Delta y_{i,j} = Y_{i,j} - Y_{i,j-1}$, $Y_{i,0} = 0$.
- Denote $\boldsymbol{\Delta Y}_i = (\Delta y_{i,1}, \ldots, \Delta y_{i,n_i})^\top$, $\boldsymbol{\Delta Y} = \left(\boldsymbol{\Delta Y}_1^\top, \cdots, \boldsymbol{\Delta Y}_I^\top\right)^\top$.

Conditional PDF of $\Delta y_{i,j}$

$$\Delta y_{i,j} \sim rIG\left(\Delta m_{i,j}^{(k)}\left(\delta_{1,i}, \delta_{2,i}, \tau_i\right), \gamma\right),$$

$$\Delta m_{i,j}^{(k)}\left(\delta_{1,i}, \delta_{2,i}, \tau_i\right) = \begin{cases} \delta_{1,i}\Delta t_{i,j} & k=1, \\ \left(\delta_{1,i} - \delta_{2,i}\right)\tau_i + \delta_{2,i} t_{i,j} - \delta_{1,i} t_{i,j-1}, & k=2, \\ \delta_{2,i}\Delta t_{i,j}, & k=3, \end{cases}$$

$\Delta t_{i,j} = t_{i,j} - t_{i,j-1}$ and $t_{i,0} = 0$, $i = 1 \ldots, I,\ j = 1, \ldots, n_i$.



(a) $\tau_i \geq t_{i,j}$ (b) $t_{i,j-1} \leq \tau_i < t_{i,j}$ (c) $\tau_i < t_{i,j-1}$

Figure 1: Three scenarios for change points and inspection time.

Conditional PDF of $\Delta y_{i,j}$

Let $\lambda_{i,j}^{(1)} = \mathcal{I}\left(\tau_i \geq t_{i,j}\right), \lambda_{i,j}^{(2)} = \mathcal{I}\left(t_{i,j-1} \leq \tau_i < t_{i,j}\right), \lambda_{i,j}^{(3)} = \mathcal{I}\left(\tau_i < t_{i,j-1}\right)$.

$$\Delta m_{i,j}\left(\delta_{1,i}, \delta_{2,i}, \tau_i\right) = \Delta m_{i,j}^{(1)}\left(\delta_{1,i}, \delta_{2,i}, \tau_i\right)^{\lambda_{i,j}^{(1)}} \times \Delta m_{i,j}^{(2)}\left(\delta_{1,i}, \delta_{2,i}, \tau_i\right)^{\lambda_{i,j}^{(2)}} \times \Delta m_{i,j}^{(3)}\left(\delta_{1,i}, \delta_{2,i}, \tau_i\right)^{\lambda_{i,j}^{(3)}}.$$

$$\begin{aligned} f_{i,j}\left(\Delta y_{i,j} \mid \delta_{1,i}, \delta_{2,i}, \tau_i, \gamma\right) = & \frac{\Delta m_{i,j}\left(\delta_{1,i}, \delta_{2,i}, \tau_i\right)}{\sqrt{2\pi}} \exp\left\{\gamma \Delta m_{i,j}\left(\delta_{1,i}, \delta_{2,i}, \tau_i\right)\right\} \Delta y_{i,j}^{-3/2} \\ & \times \exp\left\{-\frac{\left[\Delta m_{i,j}\left(\delta_{1,i}, \delta_{2,i}, \tau_i\right)\right]^2 \Delta y_{i,j}^{-1} + \gamma^2 \Delta y_{i,j}}{2}\right\}. \end{aligned}$$

Likelihood

- Let $\boldsymbol{\delta}_1 = (\delta_{1,1}, \ldots, \delta_{1,I})^\top$, $\boldsymbol{\delta}_2 = (\delta_{2,1}, \ldots, \delta_{2,I})^\top$ and $\boldsymbol{\tau} = (\tau_1, \ldots, \tau_I)^\top$.
- Denote $\boldsymbol{\eta} = \left(\boldsymbol{\delta}_1^\top, \boldsymbol{\delta}_2^\top, \gamma\right)^\top$, $\boldsymbol{\theta}_\tau = \left(\mu_\tau, \sigma_\tau^2\right)^\top$ and $\boldsymbol{\vartheta} = \left(\boldsymbol{\theta}_\tau^\top, \boldsymbol{\eta}^\top\right)^\top$.
- Given the observed data $\boldsymbol{\Delta Y}$, the likelihood function is

$$L_{obs}(\boldsymbol{\Delta Y}|\boldsymbol{\vartheta}) = \prod_{i=1}^{I} \int_{-\infty}^{\infty} \prod_{j=1}^{n_i} f_{i,j}\left(\Delta y_{i,j} \mid \delta_{1,i}, \delta_{2,i}, \tau_i, \gamma\right) g_\tau(\tau_i|\boldsymbol{\theta}_\tau) \mathrm{d}\tau_i. \quad (14)$$

Remark: Obtain a closed-form solution for the ML estimates of $\boldsymbol{\vartheta}$ is not feasible.

EM Algorithm

Log-likelihood function for the complete data

$$l_c(\boldsymbol{\Delta Y}, \boldsymbol{\tau} | \boldsymbol{\vartheta}) = \sum_{i=1}^{I} l_i\left(\boldsymbol{\theta}_\tau\right) + \sum_{i=1}^{I} \sum_{j=1}^{n_i} l_{i,j}(\boldsymbol{\eta}, \boldsymbol{\tau}), \tag{15}$$

$$l_i\left(\boldsymbol{\theta}_\tau\right) = \log g_\tau\left(\tau_i \mid \boldsymbol{\theta}_\tau\right) = -\log \sqrt{2\pi}\sigma_\tau - \frac{\left(\tau_i - \mu_\tau\right)^2}{2\sigma_\tau^2},$$

$$\begin{aligned} l_{i,j}(\boldsymbol{\eta}, \boldsymbol{\tau}) &= \log f_{i,j}\left(\Delta y_{i,j} \mid \boldsymbol{\eta}, \boldsymbol{\tau}\right) \\ &= -\log \sqrt{2\pi} + \log \Delta m_{i,j} + \gamma \Delta m_{i,j} - \frac{3}{2} \log \Delta y_{i,j} - \frac{\Delta m_{i,j}^2}{2\Delta y_{i,j}} - \frac{\gamma^2 \Delta y_{i,j}}{2}, \end{aligned}$$

and $\Delta m_{i,j} = \Delta m_{i,j}\left(\delta_{1,i}, \delta_{2,i}, \tau_i\right)$.

EM Algorithm

- E-step:

$$\begin{aligned} \boldsymbol{Q}_{(s)}(\boldsymbol{\vartheta}) &= E_{\boldsymbol{\vartheta}_{(s)}}\left[l_c(\boldsymbol{\Delta Y}, \boldsymbol{\tau}|\boldsymbol{\vartheta})\right] \\ &= \sum_{i=1}^{I} E_{\boldsymbol{\vartheta}_{(s)}}\left[l_i\left(\boldsymbol{\theta}_{\tau}\right) \mid \boldsymbol{\Delta Y}\right] + \sum_{i=1}^{I}\sum_{j=1}^{n_i} E_{\boldsymbol{\vartheta}_{(s)}}\left[l_{i,j}(\boldsymbol{\eta}, \boldsymbol{\tau}) \mid \boldsymbol{\Delta Y}\right], \end{aligned} \tag{16}$$

- M-step:

$$\boldsymbol{\vartheta}_{(s+1)} = \arg\max \boldsymbol{Q}_{(s)}(\boldsymbol{\vartheta}). \tag{17}$$

EM Algorithm

- **Step 1**. Initialize the parameters $\boldsymbol{\vartheta}$ to some random values $\boldsymbol{\vartheta}_{(\mathbf{0})}$, and setting the tolerance error ϵ.
- **Step 2**. Calculate $E_{\boldsymbol{\vartheta}_{(s)}}\left[l_i\left(\boldsymbol{\theta}_\tau\right) \mid \boldsymbol{\Delta y}\right]$ and $E_{\boldsymbol{\vartheta}_{(s)}}\left[l_{i,j}(\boldsymbol{\eta}, \boldsymbol{\tau}) \mid \boldsymbol{\Delta y}\right]$ based on the solution of the s-th iteration $\boldsymbol{\vartheta}_{(s)}$.
- **Step 3**. Calculate the solution of the $(s+1)$-th iteration $\boldsymbol{\vartheta}_{(s+1)}$ by (17).
- **Step 4**. Repeat Steps 2 and 3 until $\left|\boldsymbol{\vartheta}_{(s+1)} - \boldsymbol{\vartheta}_{(s)}\right| < \epsilon$, where $|\cdot|$ is the Euclidean distance.
- **Step 5**. The MLE of $\boldsymbol{\vartheta}$ can be obtained as $\hat{\boldsymbol{\vartheta}} = \boldsymbol{\vartheta}_{(s+1)}$.

Parametric bootstrap method

Algorithm 1: Parametric bootstrap algorithm.

Input: Point estimate $\hat{\boldsymbol{\vartheta}}$.

Output: $\mathcal{B}$ bootstrap estimates $\left\{\hat{\boldsymbol{\vartheta}}_1^*, \ldots, \hat{\boldsymbol{\vartheta}}_{\mathcal{B}}^*\right\}$.

for $b = 1$ **to** $\mathcal{B}$ **do**

  Generate $\boldsymbol{\tau}$ from $\mathcal{N}(\hat{\mu}_\tau, \hat{\sigma}_\tau^2)$;

  for $i = 1$ **to** I **do**

    for $j = 1$ **to** n_i **do**

      Generate degradation sample $\Delta\tilde{Y}_{i,j}$ from
      $rIG\left(\Delta m_{i,j}^{(k)}\left(\hat{\delta}_{1,i}, \hat{\delta}_{2,i}, \hat{\tau}_i\right), \hat{\gamma}\right), k = 1, 2, 3.$

    end

  end

  Obtain $\hat{\boldsymbol{\vartheta}}_b^*$ based on $\Delta\tilde{\boldsymbol{Y}}$ using the proposed EM algorithm.

end

Parametric bootstrap method

After acquiring the bootstrap estimates $\left\{\hat{\boldsymbol{\vartheta}}_1^*, \ldots, \hat{\boldsymbol{\vartheta}}_{\mathcal{B}}^*\right\}$, an approximate $100(1-\alpha)\%$ bootstrap confidence interval for a function of the parameters $h(\boldsymbol{\vartheta})$ is:

$$\left[h\left(\hat{\boldsymbol{\vartheta}}^*\right)_{(\alpha\mathcal{B}/2)}, h\left(\hat{\boldsymbol{\vartheta}}^*\right)_{((1-\alpha/2)\mathcal{B})}\right],$$

where $h\left(\hat{\boldsymbol{\vartheta}}^*\right)_{(b)}$ denotes the b-th statistic among $\left\{h\left(\hat{\boldsymbol{\vartheta}}^*\right)_1, \ldots, h\left(\hat{\boldsymbol{\vartheta}}^*\right)_{\mathcal{B}}\right\}$.

Bayesian analysis

$$
\begin{aligned}
&Y_i(t|\tau_i) \sim r\mathcal{IG}\left(m(t;\delta_{1,i},\delta_{2,i},\tau_i),\gamma\right),\ \tau_i \sim N\left(\mu_\tau,\sigma_\tau^2\right),\ i=1,\ldots,I,\\
&m(t;\delta_{1,i},\delta_{2,i},\tau_i) = \begin{cases}\delta_{1,i}t, & t\le\tau_i,\\ \delta_{2,i}\left(t-\tau_i\right)+\delta_{1,i}\tau_i, & t>\tau_i,\end{cases}\\
&\left(\mu_\tau,\sigma_\tau^2\right) \sim NIGa\left(\beta_\tau,\eta_\tau,\upsilon_\tau,\xi_\tau\right),\gamma\sim N(\omega,\kappa^2),\\
&\delta_{1,i}\sim N\left(\mu_1,\sigma_1^2\right),\delta_{2,i}\sim N\left(\mu_2,\sigma_2^2\right),\\
&\left(\mu_1,\sigma_1^2\right)\sim NIGa\left(\beta_1,\eta_1,\upsilon_1,\xi_1\right),\left(\mu_2,\sigma_2^2\right)\sim NIGa\left(\beta_2,\eta_2,\upsilon_2,\xi_2\right),
\end{aligned}
$$

where $NIGa(\cdot)$ denotes the normal-inverse gamma distribution.

Joint posterior distribution of $\boldsymbol{\theta}$

- Let $\boldsymbol{\theta} = \left(\boldsymbol{\vartheta}, \mu_1, \sigma_1^2, \mu_2, \sigma_2^2\right)^\top$ be the parameter vector.
- According to Bayes' theorem, the joint posterior distribution of $\boldsymbol{\theta}$ can be derived as

$$\begin{aligned}\pi(\boldsymbol{\theta} \mid \boldsymbol{\Delta Y}) \propto\ & \pi\left(\mu_\tau, \sigma_\tau^2\right) \pi\left(\mu_1, \sigma_1^2\right) \pi\left(\mu_2, \sigma_2^2\right) \pi\left(\gamma \mid \omega, \kappa\right) \pi\left(\tau \mid \mu_\tau, \sigma_\tau^2\right) \\ & \times \pi\left(\boldsymbol{\delta}_1 \mid \mu_1, \sigma_1^2\right) \pi\left(\boldsymbol{\delta}_2 \mid \mu_1, \sigma_1^2\right) f_{\Delta Y}\left(\boldsymbol{\Delta Y} \mid \boldsymbol{\delta}_1, \boldsymbol{\delta}_2, \boldsymbol{\tau}, \gamma\right).\end{aligned} \tag{18}$$

- Employ the **Gibbs sampling algorithm** to generate posterior samples of the parameters, thereby facilitating Bayesian inference.

Outline

Adaptive replacement policy

- $0 = t_{i,0} < t_{i,1} < \cdots < t_{i,j}$ are discrete inspection times
- $Y_{i,j}$ represents the observed degradation value. $Y_{i,1:j} = \{Y_{i,1}, Y_{i,2}, \ldots, Y_{i,j}\}$.
- Iteratively update estimations of model parameters and RUL distributions, $f_{S_t}(x|Y_{i,1:j})$.

Idea

1. Evaluate **candidate maintenance actions** at each inspection time point;
2. Determine **optimal or final maintenance actions** as data continues to be collected.

Policy assumption

- Maintenance is executed perfectly by replacing the system spare parts.
- An adequate supply of spare parts.
- Maintenance preparation time ϖ is usually required.

Two maintenance actions

At $t_{i,j}$, the decision maker has the option: replace the system or wait until the next inspection.

- Corrective replacement: implement if the system is found to have failed during the inspection, incurring a corrective replacement cost denoted as c_c.
- Preventive replacement: implement when it is expected that the system is nearing the failure state, incurring a preventive replacement cost denoted as c_p.

Candidate replacement time at $t_{i,j}$

$$\mathcal{T}_{i,j} = \inf_{T_{i,j}} \left\{ \int_0^{T_{i,j}-t_{i,j}} \frac{c_c}{x+t_{i,j}} f_{S_t}(x|Y_{i,1:j})dx + \int_{T_{i,j}-t_{i,j}}^{+\infty} f_{S_t}(x|Y_{i,1:j}) \frac{c_p}{T_{i,j}} dx \right\}.$$

Optimal replacement time

- As the values of $\mathcal{T}_{i,j}$ are successively updated,

$$\mathcal{T}_i^* = \inf_{t_{i,j}} \{\mathcal{T}_{i,j} - t_{i,j} \leq \varpi\}. \tag{19}$$

Performance evaluation

- Consider a set of I systems, each of which operates for a single cycle.
- Let $\mathbb{X}_i = \min\{\mathcal{T}_i^*, \mathcal{T}_i^{\mathsf{f}}\}$, where $\mathcal{T}_i^*$ represents predicted optimal maintenance time, and $\mathcal{T}_i^{\mathsf{f}}$ represents actual failure time.

Actual cost rate of the i-th system

$$CR_i = \begin{cases} \dfrac{c_p}{\mathcal{T}_i^*}, & \mathbb{X}_i = \mathcal{T}_i^*, \\ \dfrac{c_c}{\mathcal{T}_i^{\mathsf{f}}}, & \mathbb{X}_i = \mathcal{T}_i^{\mathsf{f}}. \end{cases} \tag{20}$$

Average cost rate for all systems

$$\overline{CR} = \frac{\sum_{i=1}^{I} \mathbb{X}_i \cdot CR_i}{\sum_{i=1}^{I} \mathbb{X}_i}. \tag{21}$$

Algorithm 3: RUL-based adaptive replacement policy

Input: $y, c_c, c_p, \varpi, \mathcal{D}$.

Output: $\mathcal{T}_i^*$, $CR_i, i = 1, \ldots, I$, and $\overline{CR}$.

for $i = 1$ **to** I **do**
    while *no maintenance performed* **do**
        if *the system is operational* **then**
            Collect new inspection data $Y_{i,j}$;
            Update estimation of model parameters via EM algorithm or Bayesian method in Section 3;
            Compute RUL distribution $\{f_{S_t}(x|Y_{i,1:j})\}_{x=0}^{+\infty}$ using (14) ;
            Determine $\mathcal{T}_{i,j}$ by (27), and find $\mathcal{T}_i^*$ by (28);
            if $t_{i,j} = \mathcal{T}_i^*$ **then**
                Preventive maintenance.
            end
        end
        else
            Corrective maintenance;
            Set $\mathcal{T}_i^{\mathrm{f}} = t_{i,j}$.
        end
        $i = i + 1$
    end
    Compute CR_i by (29).
end
Compute $\overline{CR}$ by (30).

Benchmark policies

i) **Classical replacement policy (CRP)**: preventive maintenance time is determined by the system's mean time to failure $\bar{\mathcal{T}}^F$.

ii) **Ideal replacement policy (IRP)**: the assumption of perfect predicted failure time $\mathcal{T}_i^P$.

Outline

Simulation settings

- (I) $I = 5$ and $n_i = 20$; (II) $I = 5$ and $n_i = 40$; (III) $I = 8$ and $n_i = 20$.
- $\delta_1 \sim N(15, 1), \delta_2 \sim N(4, 1)$, and $\tau = N(10, 1)$.
- 500 simulated samples are repeatedly generated from each scenario.
- ML method: the point estimates are calculated by the EM algorithm, corresponding interval estimates are calculated by parametric bootstrap method with $\mathcal{B} = 500$.
- HB method: the posterior samples of $\boldsymbol{\theta}$ are generated via the ARMS-Gibbs algorithm. To obtain posterior samples for each scenario, we initiate a burn-in period comprising $\mathcal{L} = 5000$ iterations.
- Indexes of assessing different methods: relative bias (RB), rooted mean squared error (RMSE) and 95% coverage probability (CP).

Table 1: Parameter estimation from HB and ML methods for two scenarios.

Scen.	Meth.	Stat.	$\delta_{1,1}$	$\delta_{1,2}$	$\delta_{1,3}$	$\delta_{1,4}$	$\delta_{1,5}$	$\delta_{2,1}$	$\delta_{2,2}$	$\delta_{2,3}$	$\delta_{2,4}$	$\delta_{2,5}$	γ
		RB	0.024	0.029	-0.007	0.015	0.012	-0.026	0.019	0.023	0.056	0.003	0.011
	HB	RMSE	1.326	1.363	1.357	1.332	1.330	0.422	0.424	0.476	0.422	0.431	0.168
		CP	0.956	0.953	0.946	0.953	0.957	0.941	0.925	0.900	0.928	0.926	0.964
		RB	0.057	0.039	0.040	0.057	0.050	0.065	0.071	0.057	0.078	0.060	0.057
I	MLE	RMSE	1.315	1.381	1.302	1.401	1.508	0.641	0.645	0.576	0.667	0.739	0.308
		CP	0.889	0.922	0.878	0.900	0.833	0.922	0.922	0.900	0.889	0.867	0.811
		Stat.	τ_1	τ_2	τ_3	τ_4	τ_5						
		RB	0.002	0.001	0.002	0.001	-0.009						
	HB	RMSE	0.248	0.224	0.240	0.191	0.243						
		CP	0.915	0.937	0.937	0.961	0.961						
Scen.	Meth.	Stat.	$\delta_{1,1}$	$\delta_{1,2}$	$\delta_{1,3}$	$\delta_{1,4}$	$\delta_{1,5}$	$\delta_{2,1}$	$\delta_{2,2}$	$\delta_{2,3}$	$\delta_{2,4}$	$\delta_{2,5}$	γ
		RB	-0.005	0.007	0.023	0.011	-0.005	-0.019	0.000	0.016	0.000	0.012	0.001
	HB	RMSE	1.068	1.011	1.065	1.015	1.044	0.349	0.283	0.275	0.355	0.332	0.124
		CP	0.930	0.945	0.950	0.944	0.927	0.902	0.925	0.947	0.885	0.902	0.914
		RB	0.036	0.035	0.017	0.032	0.039	0.029	0.041	0.036	0.025	0.042	0.039
II	MLE	RMSE	0.944	1.010	0.880	0.900	0.985	0.331	0.358	0.323	0.328	0.346	0.150
		CP	0.905	0.890	0.905	0.920	0.900	0.895	0.890	0.930	0.930	0.920	0.865
		Stat.	τ_1	τ_2	τ_3	τ_4	τ_5						
		RB	0.002	0.000	-0.001	0.003	-0.004						
	HB	RMSE	0.225	0.214	0.218	0.185	0.189						
		CP	0.951	0.941	0.929	0.966	0.942						

Outline

Lithium-ion batteries

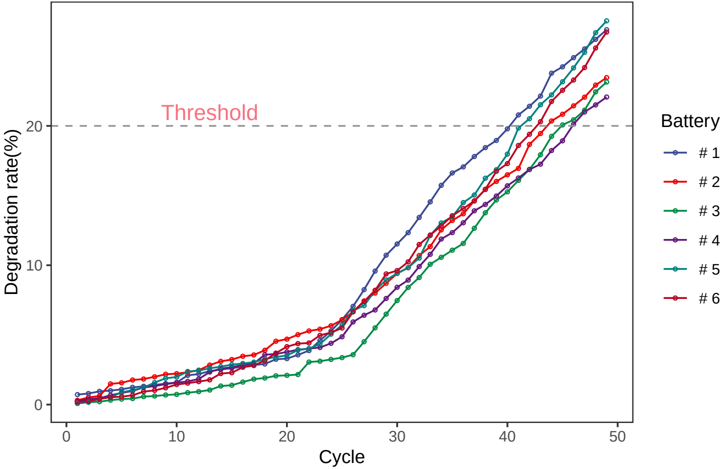


Figure 2: Capacity degradation data of 6 lithium batteries.

Parameter Estimation by two-phase rIG Model

Table 2: Parameter estimation based on the proposed model.

		HB			ML				HB			ML	
		β_1	β_2	τ	β_1	β_2			β_1	β_2	τ	β_1	β_2
	2.5%	0.422	2.198	22.257	0.497	2.511		2.5%	0.467	1.993	24.151	0.561	2.12
# 1	Mean	0.532	2.516	23.187	0.510	2.632	# 4	Mean	0.583	2.291	25.008	0.576	2.221
	97.5%	0.645	2.851	24.664	0.518	2.713		97.5%	0.703	2.595	26.060	0.587	2.288
	2.5%	0.523	2.013	24.365	0.638	2.113		2.5%	0.495	2.162	23.184	0.624	2.382
# 2	Mean	0.653	2.312	25.336	0.658	2.215	# 5	Mean	0.621	2.472	24.003	0.642	2.496
	97.5%	0.785	2.615	26.557	0.670	2.282		97.5%	0.752	2.809	25.370	0.654	2.572
	2.5%	0.336	2.161	26.316	0.405	2.412		2.5%	0.464	2.130	24.722	0.559	2.324
# 3	Mean	0.428	2.487	26.761	0.414	2.531	# 6	Mean	0.577	2.443	25.583	0.574	2.440
	97.5%	0.518	2.831	27.381	0.420	2.61		97.5%	0.697	2.769	26.306	0.585	2.517

Table 3: RMSE and RB results for different models.

Model	Training(30)		Prediciton (19)		Overall	
	RMSE	RB	RMSE	RB	RMSE	RB
Proposed	0.448	0.248	1.538	0.060	1.020	0.175
Linear	3.476	1.442	3.685	0.156	3.558	0.943
Power	2.057	0.568	2.475	0.113	2.229	0.391
Exp	0.908	0.313	1.611	0.065	1.230	0.217
Duan	0.434	0.239	1.976	0.075	1.276	0.175

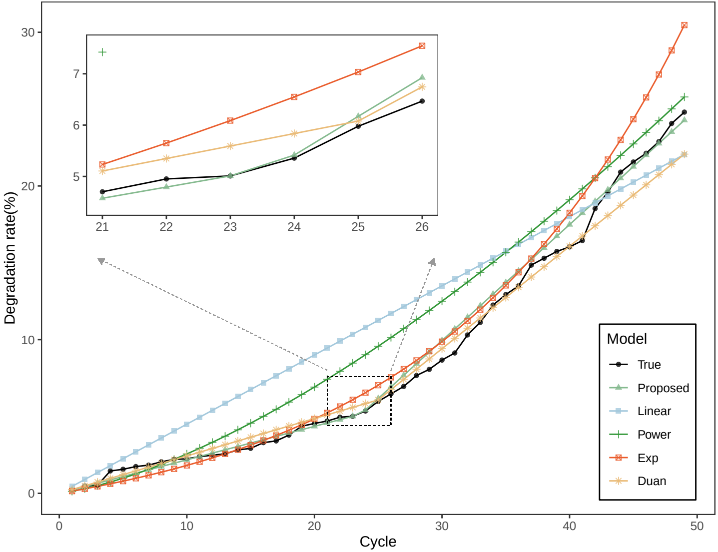


Figure 3: Degradation path training and prediction results for battery #2 using different methods, with a zoomed-in view of the potential change point locations.

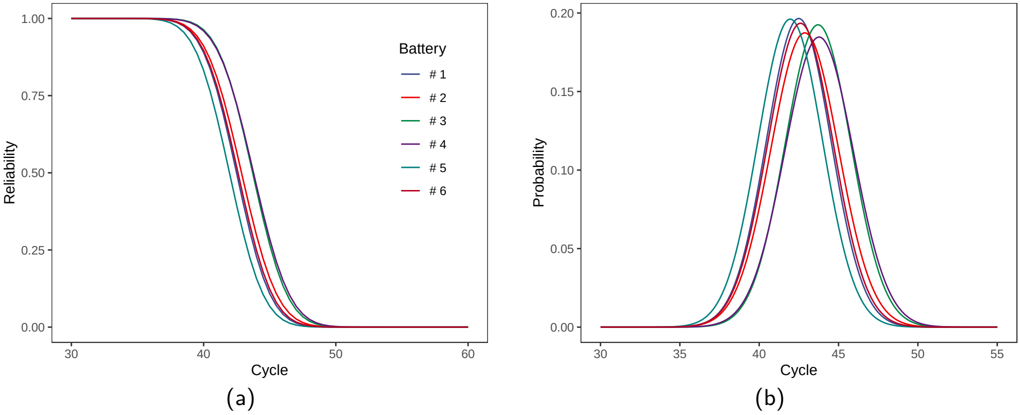


Figure 4: Reliability and density functions of failure time based on HB method.

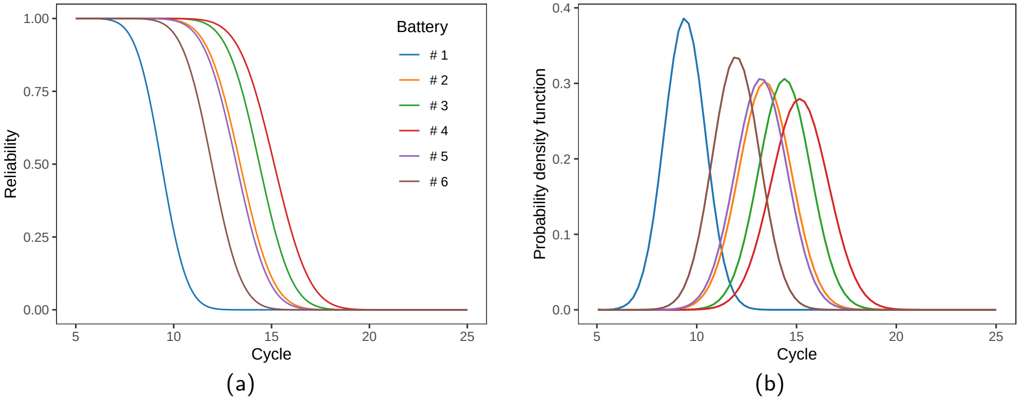


Figure 5: Reliability and density functions of RUL based on HB method.

RUL-based adaptive maintenance policy

Table 4: Candidate replacement time at consecutive data-acquire epochs.

Cycle	Battery #2			Battery #3		
	Real RUL	MRL	$\mathcal{T}_{2,j}$	Real RUL	MRL	$\mathcal{T}_{3,j}$
31	12	13.865	41.3	13	13.228	40.9
33	10	11.219	41.0	11	10.278	40.4
35	8	7.624	39.9	9	8.389	40.7
37	6	5.986	40.6	7	6.884	41.5
39	4	4.040	41.1	5	4.206	41.2
41	2	2.764	42.1	3	2.318	42.0
43	-	0.062	44.0	1	0.380	44.0

Table 5: Maintenance cost rates for 6 batteries under the adaptive replacement policy.

Battery	FC	TS			Linear			Power			Exp		
		$\mathcal{T}_i^*$	Ac	CR	$\mathcal{T}_i^*$	Ac	CR	$\mathcal{T}_i^*$	Ac	CR	$\mathcal{T}_i^*$	Ac	CR
1	40	36	P	5.556	36	P	5.556	38	P	5.263	35	P	5.714
2	43	42	P	4.762	40	P	5.000	43	P	4.651	40	P	5.000
3	44	41	P	4.878	41	P	4.878	43	P	4.651	41	P	4.878
4	45	43	P	4.651	41	P	4.878	-	C	13.333	41	P	4.878
5	41	40	P	5.000	39	P	5.128	-	C	14.634	38	P	5.263
6	42	41	P	4.878	40	P	5.000	42	P	4.762	39	P	5.128

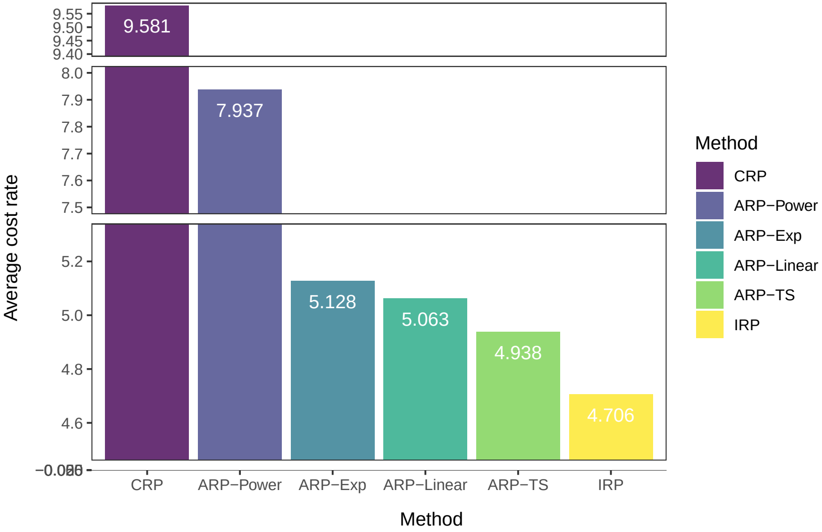


Figure 6: Average cost rate for each policy.

Thanks!